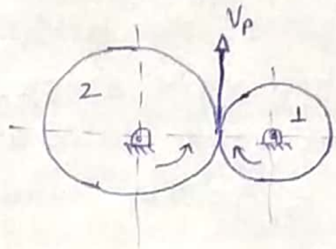
GEARS

Introduction → If power transmitted between two shafts is small, motion between them can be obtained by using two plain cylinders or discs.

If there is no slip of one surface relative to the other, such wheels are termed as friction wheels.

However, as the power transmitted increases, slip occurs b/w the discs.



Assuming no slipping,

linear velocity

$$V_p = \omega_1 r_1 = \omega_2 r_2$$

$$2\pi N_1 r_1 = 2\pi N_2 r_2$$

or $$\frac{\omega_1}{\omega_2} = \frac{N_1}{N_2} = \frac{r_2}{r_1}$$

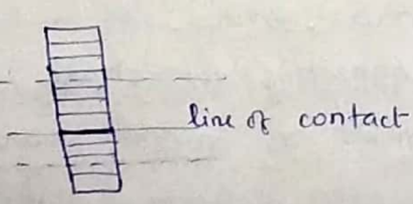
N = No. of revolution (rpm)
ω = angular velocity (rad/s)
r = radius of disc

To transmit a definite motion of one disc to the other or to prevent slip b/w the surfaces, projections and recesses on the two discs can be made which can mesh with each other. This leads to the formation of teeth on the discs and the motion b/w the surfaces changes from rolling to sliding. The discs with teeth are known as gear or gear wheels.

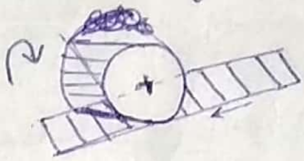
Classification of Gears → Gears can be classified according to the relative positions of their shaft axes as follows:

(1) **Parallel Shafts**:-

Spur Gears → They have straight teeth parallel to the axes & thus are not subjected to axial thrust due to tooth load.

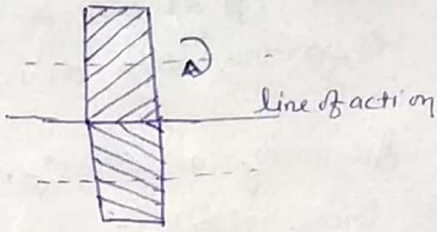


Spur Rack & Pinion → Spur rack is a special case of a spur gear where it is made of infinite diameter so that the pitch surface is a plane. This combination converts rotary motion into translatory motion, or vice-versa.



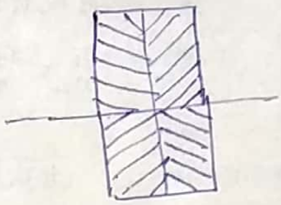
Helical Gear or Helical Spur Gear →

In helical gears, the teeth are curved, each being helical in shape. Two mating gears have the same helix angle, but have teeth of opposite hands.



Double-helical & Herringbone Gears →

A double helical gear is equivalent to a pair of helical gears, one having right hand helix and the other a left-hand helix.

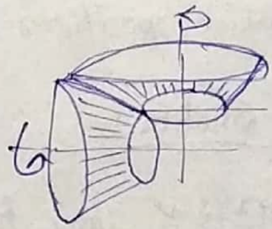


If the left and the right inclination of a double-helical gear meet at a common apex & there is no groove, the gear is known as herringbone gear.

(2) **Intersecting Shaft :-** Kinematically, the motion b/w two intersecting shafts is equivalent to the rolling of two cones, are known as Bevel gears.

Straight Bevel Gears →

The teeth are straight, radial to the point of intersection of the shaft axes & vary in cross section throughout their length.



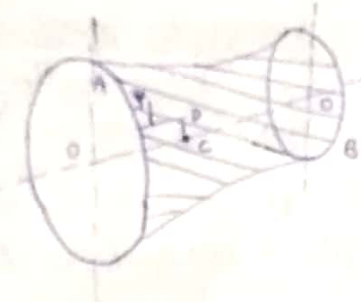
Gears of the same size & connecting two shafts at right angles to each other are known as mitre gears.

Spiral Bevel Gears →

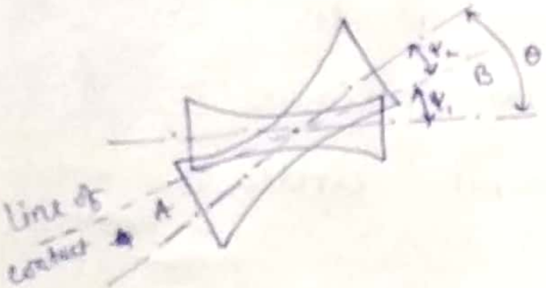
When the teeth of a bevel gear are inclined at an angle to the face of the bevel, they are known as spiral bevels or Helical bevels.

Uses → differential of an automobile.

(3) Skew-Shafts :- non-parallel, non-intersecting.



Observe a hyperboloid, It is a surface of revolution generated by a skew line AB revolving around an axis O-O in another plane, keeping the angle ψ_2 b/w them as constant.



$\theta = \psi_1 + \psi_2$

If two hyperboloids rotate on their respective axes, the motion b/w them would be a combination of rolling (normal to the line of contact) & sliding action (parallel to the line of action). Teeth are cut on the hyperboloid surfaces parallel to the line of contact to form gears.

Worm Gears → Worm gears are special case of spiral gears, in which the larger wheel, usually, has a hollow or concave shaped such that a portion of the pitch diameter of the other gear is enveloped on it.

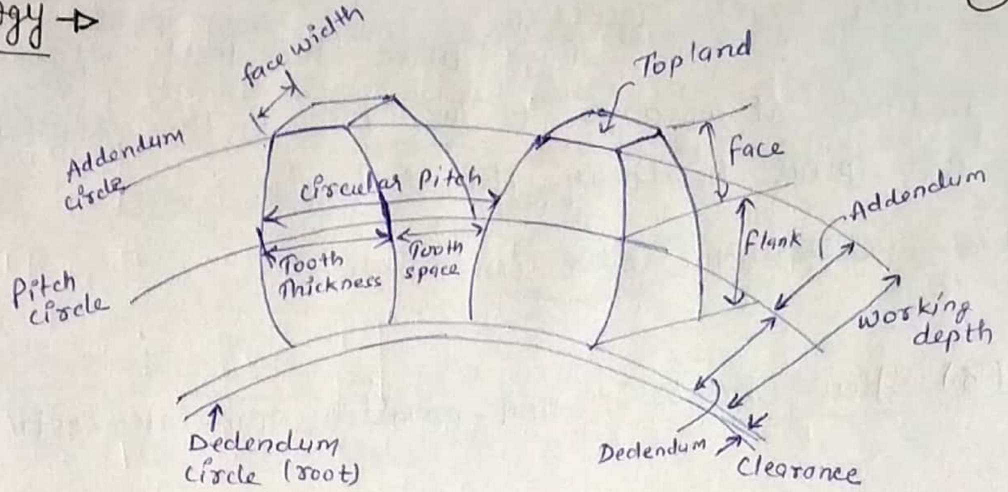
Worm gears are made in following forms:-

(1) Non throated → The contact between the teeth is concentrated at a point

(2) Single-throated → Gear teeth are curved to envelop the worm. There is line contact b/w the teeth

(3) Double-throated → There is area contact b/w the teeth. A worm may be cut with a single or a multiple thread cutter.

Gear Terminology →



(a) Pitch Cylinders → Pitch cylinders of a pair of gears in mesh are the imaginary friction cylinders, which by pure rolling together, transmit the same motion as the pair of gears.

(b) Pitch circle → It is the circle corresponding to a section of the equivalent pitch cylinder by a plane normal to the wheel axes.

(c) Pitch Diameter → It is the diameter of the pitch cylinder.

(d) Pitch Surface → It is the surface of the pitch cylinder.

(e) Pitch point → The point of contact of two pitch circles is known as the pitch point.

(f) Line of Centres → A line through the centres of rotation of a pair of mating gears is the line of centres.

(g) Addendum Circle → It is a circle passing through the tips of teeth.

(h) Addendum → It is the radial height of a tooth above the pitch circle. Its standard value is one module.

(i) Dedendum or Root Circle → It is a circle passing through the roots of the teeth.

(j) Dedendum → It is the radial depth of a tooth below the pitch circle. Its standard value is 1.157 m.

(k) Clearance → Radial difference b/w the addendum & dedendum of a tooth.

Thus, Addendum circle dia = d + 2m

Dedendum circle dia = d − 2×1.157m

Clearance = 1.157m − m

= 0.157m

(a) Full Depth of Teeth → It is the total radial depth of the tooth space.

Full Depth = Addendum + Dedendum

(b) Working Depth of Teeth → The maximum depth to which a tooth penetrates into the tooth space of the mating gear.

Working Depth = sum of addendums of the two gears.

(c) Space width → It is the width of the tooth space along the pitch circle.

(d) Tooth Thickness → It is the thickness of the tooth measured along the pitch circle.

(e) Backlash → It is the difference b/w the space width & the tooth thickness along the pitch circle.

Backlash = space width − Tooth thickness

(f) Face width → The length of the tooth parallel to the gear axis.

(g) Top Land → It is the surface of the top of the tooth.

(h) Bottom Land → The surface of the bottom of the tooth b/w the adjacent fillets.

(i) Face → Tooth surface b/w the pitch circle & the top land.

(j) Flank → Tooth surface b/w the pitch circle & the bottom land including fillet

(k) Fillet → It is the curved portion of the tooth flank at the root circle.

Pitch is defined as follows:-

(a) Circular Pitch (p) → It is the distance measured along the circumference of the pitch circle from a point on one tooth to corresponding point on the adjacent tooth.

$$\boxed{p = \frac{\pi d}{T}}$$

where, p → circular pitch
d → pitch diameter
T → Numbe of Teeth

The angle subtended by the circular pitch at the centre of the pitch circle is known as the pitch angle (γ).

(b) Diameteral Pitch (P) → It is the number of teeth per unit length of the pitch circle diameter in inches.

$$\boxed{P = \frac{T}{d}}$$

(c) Module (m) → It is the ratio of the pitch diameter in mm to the number of teeth.

$$\boxed{m = \frac{d}{T}}$$

Also,

$$p = \frac{\pi d}{T} = \pi m$$

★ Pitch of two mating gears must be same.

(d) Gear Ratio (G) → It is the ratio of the number of teeth on the gear to that on the pinion.

$$\boxed{G = \frac{T}{t}}$$

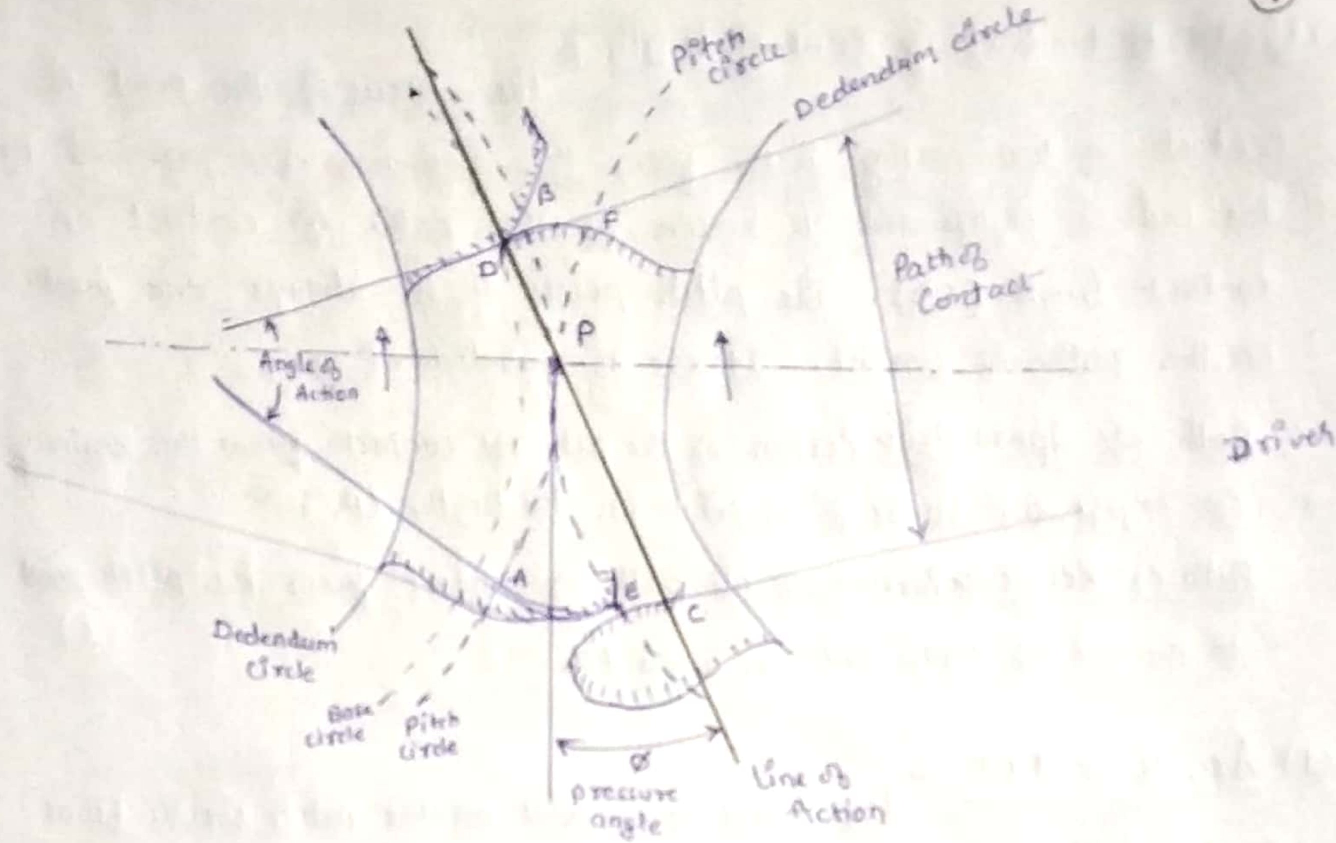
where, T = No. of teeth on gear
t = No. of teeth on pinion

(e) Velocity Ratio (VR) → The velocity Ratio is defined as the ratio of the angular velocity of the follower to the angular velocity of the driving gear.

Let d = pitch diameter
ω = angular velocity (rad/s)
N = angular velocity (rpm)
T = No. of teeth

1 → driver
2 → follower

$$VR = \frac{\text{angular velocity of follower}}{\text{angular velocity of driver}} = \frac{\omega_2}{\omega_1} = \frac{N_2}{N_1} = \frac{d_1}{d_2} = \frac{T_1}{T_2}$$



(a) Line of Action or Pressure line → The force, which the driving tooth exerts on the driven tooth, is along a line from the pitch point to the point of contact of the two teeth. This line is also the common normal at the point of contact of the mating gears and is known as the line of action or pressure line.

(b) Pressure Angle or Angle of Obliquity (ø) → The angle b/w the pressure line and the common tangent to the pitch circles is known as the pressure angle or the angle of obliquity.

For more power transmission & lesser pressure on the bearings, the pressure angle must be kept small. Standard pressure angles are 20° & 25°. Gears with 14.5° pressure angles have become almost obsolete (old).

(c) Path of Contact or Contact Length →

The locus of the point of contact of two mating teeth from the beginning of engagement to the end of engagement is known as the path of contact or contact length (CD). The pitch point P is always one point on the path of contact. It can be subdivided as:

Path of Approach → Portion of the path of contact from the begining of engagement to the pitch point i.e, the length CP.

Path of Recess → Portion of the path of contact from the pitch point to the end of engagement, ie, length PD.

(d) Arc of Contact →

The locus of a point on the pitch circle from the beginning to the end of engagement of two mating gears is known as the arc of contact. APB or EPF is the arc of contact.

Arc of Approach → It is the portion of the arc of contact from the beginning of engagement to the pitch point. i.e. length AP or EP

Arc of Recess → The portion of the arc of contact from the pitch point to the end of engagement is the arc of recess i.e., length PB or PF.

(e) Angle of Action → (δ) It is the angle turned by a gear from the beginning of engagement to the end of ~~enage~~ engagement of a pair of teeth, i.e. the angle turned by arcs of contact of respective gear wheels.

$$\boxed{\delta = \alpha + \beta}$$

$\alpha \rightarrow$ angle of approach
$\beta \rightarrow$ angle of recess

(f) Contact Ratio → It is the angle of action divided by pitch angle.

$$\boxed{\text{contact Ratio} = \frac{\delta}{\gamma} = \frac{\alpha+\beta}{\gamma}}$$

Also,

$$\boxed{\text{contact Ratio} = \frac{\text{Arc of contact}}{\text{Circular pitch}}}$$

Ex:- Two spur gears have a velocity ratio of 1/3. The driven gear has 72 teeth of 8 mm module and rotates at 300 rpm. Calculate the number of teeth & the speed of the driver. What will be the pitch line velocities?

Sol:- Given,

$T_2 = 72$

$VR = \frac{1}{3}$

$N_2 = 300\,rpm$

$m = 8\,mm$

(i) $VR = \frac{N_2}{N_1} = \frac{T_1}{T_2} = \frac{1}{3}$

$\frac{300}{N_1} = \frac{1}{3}$

$\boxed{N_1 = 900\,rpm}$

Also, $\frac{T_1}{72} = \frac{1}{3}$

$\boxed{T_1 = 24}$

(ii) Pitch line velocity,

$V_p = \omega_1 r_1$ (or) $\omega_2 r_2$

$= 2\pi N_1 \times \frac{d_1}{2}$

$= 2\pi N_1 \times \frac{mT_1}{2}$ $\quad (m = \frac{d}{T})$

$= 2\pi \times 900 \times \frac{8 \times 24}{2}$

$= 542867\,mm/minute$

$= 9047.8\,mm/s$

$V_p = 9.0478\,m/s$

Ex:-

The number of teeth of a spur gear is 30 and it rotates at 200 rpm. What will be its circular pitch and the pitch line velocity if it has a module of 2 mm?

Sol:- Given, $T = 30$,

$m = 2mm$

$N = 200\,rpm$

Pitch $p = \pi m$

$p = \pi \times 2$

$\boxed{p = 6.28\,mm}$

Pitch line velocity $V_p = \omega r$

$= 2\pi N \times \frac{d}{2}$

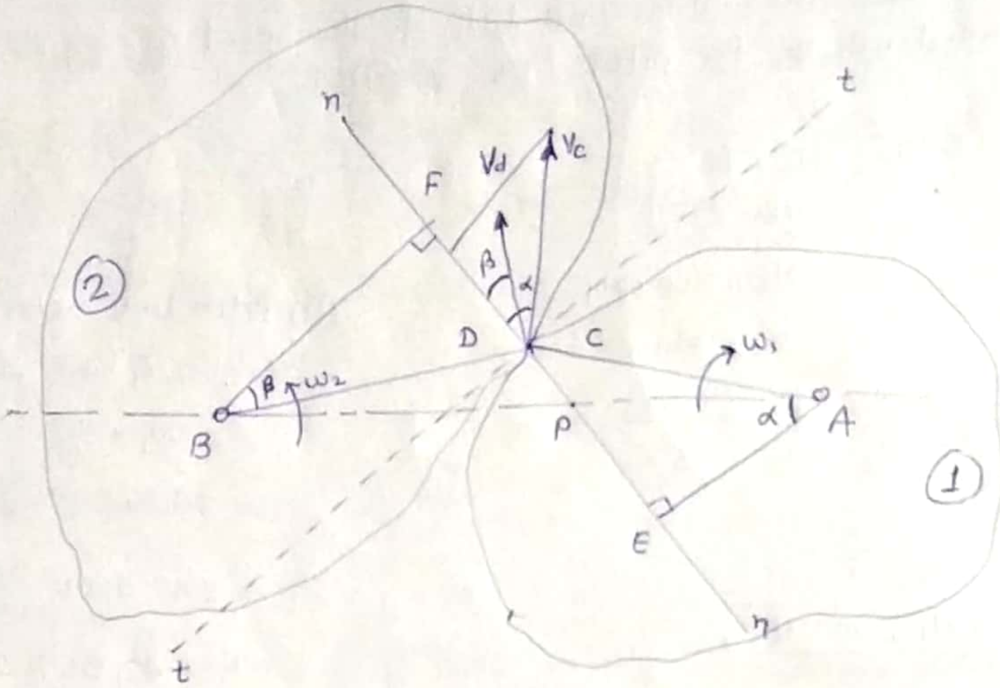
$= 2\pi N \times \frac{mT}{2}$

$= \pi \times 200 \times 2 \times 30$

$V_p = 37699\,mm/min$

$\boxed{V_p = 628.3\,mm/s}$

Law of Gearing →



A point C on the tooth profile of the gear 1 is in contact with a point D, on the tooth profile of the gear 2.

The two curves in contact at points C or D must have a common normal at the point (n-n).

Let ω_1 = instantaneous angular velocity of gear 1 (Clockwise)

ω_2 = — " ——— " ——— " ——— gear 2 (counter clockwise)

V_c = Linear velocity of C

V_d = linear velocity of D

Then, $V_c = \omega_1 \cdot AC$ in a direction $\perp$ to AC or at an angle α to n-n.

$V_d = \omega_2 \cdot BD$ in a direction $\perp$ to BD or at an angle β to n-n.

Component of V_c along n-n $= V_c \cos\alpha$

component of V_d along n-n $= V_d \cos\beta$

Relative motion along n-n $= V_c \cos\alpha - V_d \cos\beta$

Draw perpendiculars AE & BF on n-n from points A & B respectively.

Then $\angle CAE = \alpha$ & $\angle DBF = \beta$.

For proper contact

$$V_c \cos\alpha - V_d \cos\beta = 0$$

$$\omega_1 \cdot AC \cos\alpha - \omega_2 \cdot BD \cos\beta = 0$$

$$\omega_1 \cdot AC \cdot \frac{AE}{AC} - \omega_2 \cdot BD \cdot \frac{BF}{BD} = 0$$

$$\omega_1 \cdot AE - \omega_2 \cdot BF = 0$$

$$\frac{\cancel{\omega_1}}{\cancel{\omega_2}} = \frac{\cancel{AE}}{\cancel{BA}} = \frac{\cancel{BP}}{\cancel{AP}}$$

[$\because \Delta AEP$ & ΔBEP are similar]

$$\frac{\omega_1}{\omega_2} = \frac{BF}{AE} = \frac{BP}{AP}$$

Therefore, "The common normal at the point of contact of the two teeth should always pass through a fixed point P which divides the line of centres in the inverse ratio of angular velocities of two gears."

For constant angular velocity ratio of the two gears, the common normal at the point of contact of the two mating teeth must pass through the pitch point.

Velocity of Sliding → If the curved surface of the two gears 1 & 2 are to remain in contact, one can have a sliding motion relative to the other along the common tangent t-t at C or D.

Component of V_c along t-t $= V_c \sin\alpha$

component of V_d along t-t $= V_d \sin\beta$

Velocity of sliding $= V_c \sin\alpha - V_d \sin\beta$

$$= \omega_1 AC \cdot \frac{EC}{AC} - \omega_2 \cdot BD \cdot \frac{FD}{BD}$$

$$= \omega_1 \cdot EC - \omega_2 \cdot FD$$

$$= \omega_1 (EP + PC) - \omega_2 (FP - PD)$$

$$= \omega_1 EP + \omega_1 PC - \omega_2 FP + \omega_2 PC$$

[PD=PC → C & D are coinside points]

$$= (\omega_1 + \omega_2) PC + \omega_1 EP - \omega_2 FP$$

$$= (\omega_1 + \omega_2) PC + \cancel{\omega_1 FP} - \cancel{\omega_1 FP}$$

[$\omega_1 EP = \omega_2 FP$]- last eqn

$$= (\omega_1 + \omega_2) PC.$$

Velocity of Sliding = Sum of angular velocities X distance b/w pitch point & point of contact.

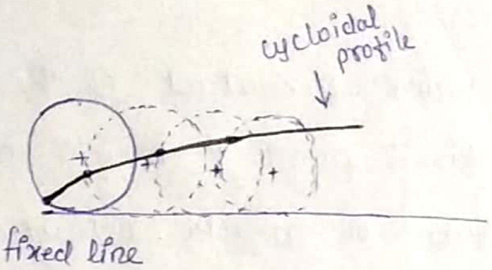
Forms of Teeth → 1. Cycloidal profile teeth
2. Involute profile teeth

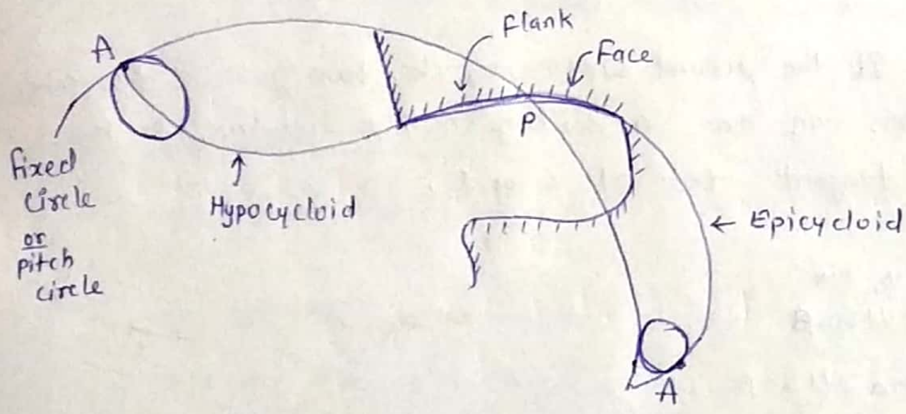
(1.) Cycloidal Profile Teeth → (By nature conjugate)

A cycloidal is the locus of a point on the circumference of a circle that rolls without slipping on a fixed straight line.

↳ epicycloidal → ~~point~~ locus of point rolls on circumference of another circle.

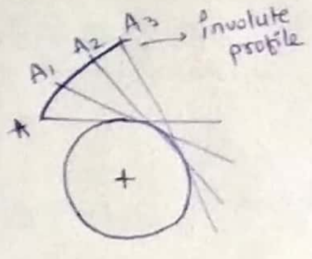
↳ Hypocycloidal → Locus of point rolls inside the circumference of another circle.

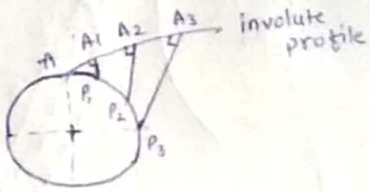




1. per tooth cost is more
2. Flank (wide) → stronger tooth
3. Convex-Concave connection → less wear → more life (5 time more than involute)

(2.) **Involute Profile Teeth** → An involute is defined as the locus of a point on a straight line which rolls without slipping on the circumference of a circle.

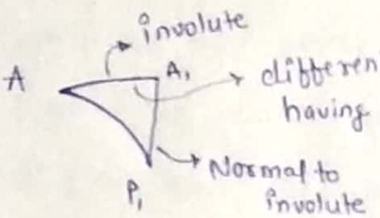




$Arc(AP_1) = A_1P_1$

$Arc(AP_2) = A_2P_2$

$Arc(AP_3) = A_3P_3$



"Normal drawn at any point on involute curve will become automatically tangent to its base circle.

Ex:- The following data relate to two meshing gears,
Velocity Ratio = $\frac{1}{3}$, module = 4 mm, Pressure angle = 20°, centre distance = 200 mm.
Determine the number of teeth and the base circle radius of gear wheel.

Sol:- Given, $VR = \frac{1}{3}$, $\phi = 20°$, $m = 4$ mm $C = 200$ mm

(i) $VR = \frac{N_2}{N_1} = \frac{1}{3} = \frac{T_1}{T_2}$

$T_2 = 3T_1$

$C = \frac{d_1 + d_2}{2} = \frac{m(T_1 + T_2)}{2}$

$200 = \frac{4(T_1 + 3T_1)}{2}$

$\boxed{T_1 = 25}$ $\quad T_2 = 3 \times 25$

$\boxed{T_2 = 75}$

(ii) $d_2 = mT_2$

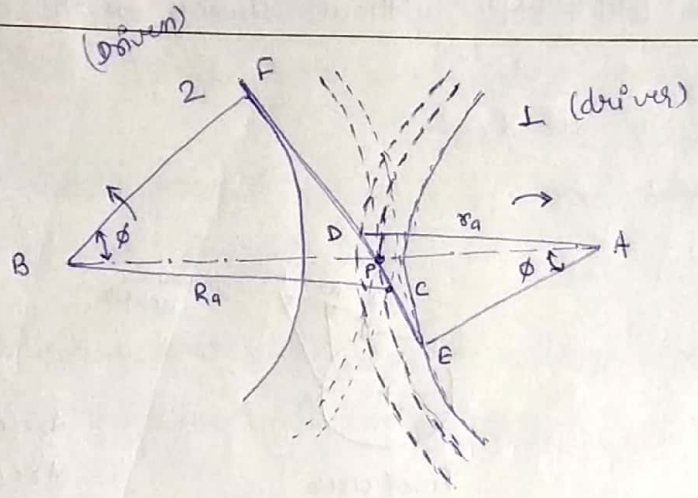
$= 4 \times 75 = 300$ mm

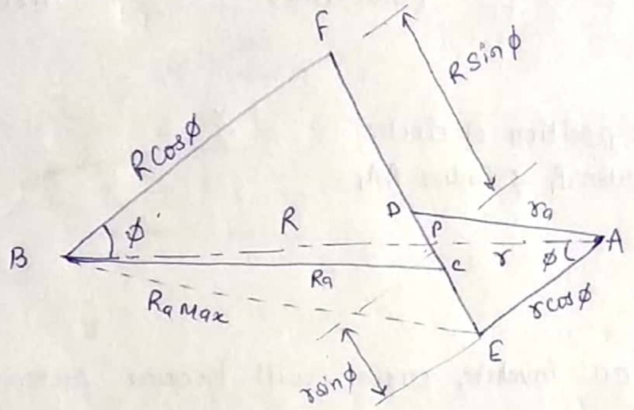
Base circle radius, $d_{b2} = \frac{d_2}{2} \cos\phi$

$= \frac{300}{2} \times \cos 20°$

$= 141$ mm

Path of contact →





$CF^2 = R_a^2 - R^2\cos^2\phi$

$DE^2 = r_a^2 - r^2\cos^2\phi$

Let r = pitch circle radius of pinion

R = pitch circle radius of wheel

r_a = addendum circle radius of pinion

R_a = addendum circle radius of wheel

Path of contact = path of approach + path of recess

$$CD = CP + PD$$

$$= (CF - PF) + (DE - PE)$$

$$= \left[\sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi\right] + \left[\sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi\right]$$

$$CD = \sqrt{R_a^2 - R^2\cos^2\phi} + \sqrt{r_a^2 - r^2\cos^2\phi} - (R+r)\sin\phi$$

Arc of Contact →

The arc of contact is the distance travelled by a point on either pitch circle of the two wheels during the period of contact of pair of teeth.

$$\boxed{\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\phi}}$$

$$\boxed{\text{Contact Ratio} = \frac{\text{Arc of contact}}{\text{Pitch circle}}}$$

Ex:- Each of two gears in a mesh has 48 teeth and a module of 8 mm. The teeth are of 20° involute profile. The arc of contact is 2.25 times the circular pitch. Determine the addendum.

Sol:- Given,

$\phi = 20°$

$t = T = 48$

$m = 8\,mm$

$R = r = \frac{mT}{2} = \frac{8 \times 48}{2} = 192\,mm$ $(R_a = r_a)$

Arc of contact $= 2.25 \times$ Circular pitch

$= 2.25 \times \pi m$

$= 2.25 \times \pi \times 8$

$= 56.55\,mm$

Path of contact $=$ Arc of contact $\times \cos\phi$

$= 56.55 \times \cos 20°$

$= 53.14\,mm$

Path of contact $= \left(\sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi\right) + \left(\sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi\right)$

$53.14 = \left(\sqrt{R_a^2 - 192^2 \cdot \cos^2 20°} - 192\sin 20°\right) + \left(\sqrt{R_a^2 - 192^2 \cdot \cos^2 20°} - 192\sin 20°\right)$

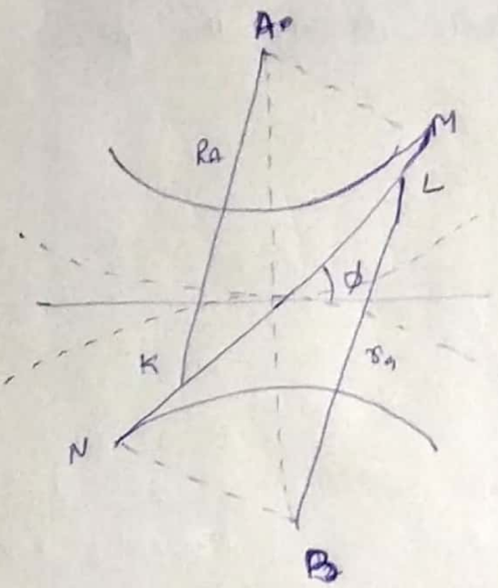
$53.14 = 2\left(\sqrt{R_a^2 - 192^2 \cdot \cos^2 20°} - 192\sin 20°\right)$

$R_a = 202.6\,mm$

Addendum $= R_a - R \Rightarrow 202.6 - 192$

Addendum $= 10.6\,mm$

Interference →



If $r_a > BM$

- Involute tip of pinion will touch non-involute flank portion of gear.
- Involute, non-involute connection
- Law of gearing is not be satisfied
- Involute tip of Pinion will remove some material from non-involute flank portion of gear
- This is called interference.

"Mating of two non-conjugate (non-involute) teeth is known as interference."

Methods to prevent Interference →

(1) Under cut gears → Under cut is provided by cutting tool at the time of manufacturing. Strength become less.

(2) Increase pressure angle → limitation → 20° - 25°

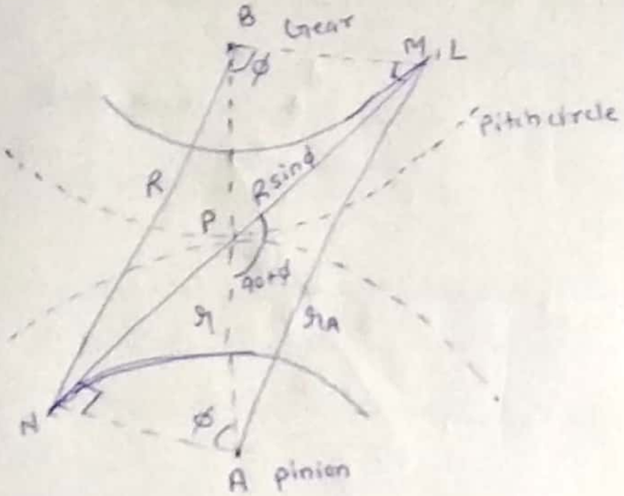
(3) Stubbing the teeth → (cut the top land)
- Addendum ↓
- Addendum circle radius ↓

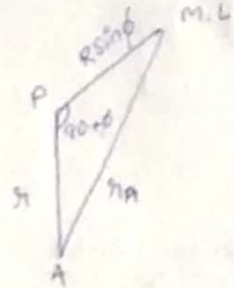
} interference ↓ — Arc of cont. ↓, Contact Ratio ↓

(4) Increase number of Teeth →
- Addendum ↓
- Addendum circle Radius ↓

} interference ↓ — contact Ratio ↑

Minimum No. of Teeth Requirement on the Pinion/Gear in order to avoid interference →





Apply Cosine Rule

$$r_A^2 = r^2 + R^2 \sin^2\phi - 2rR\sin\phi \cos(90+\phi)$$

$$r_A^2 = r^2 + (R^2 + 2r\cdot R)\sin^2\phi$$

$$r_A^2 = r^2\left[1 + \frac{R}{r}\left(\frac{R}{r}+2\right)\sin^2\phi\right]$$

$$r_A^2 = r^2\left[1 + G(G+2)\sin^2\phi\right] \qquad \left\{\because \frac{R}{r} = G\right\}$$

$$r_A = r\sqrt{[1 + G(G+2)\sin^2\phi]}$$

$$\text{Addendum (Pinion)} = r_A - r$$

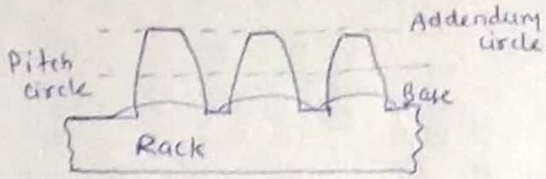
$$= r\left[\sqrt{1+G(G+2)\sin^2\phi} - 1\right]$$

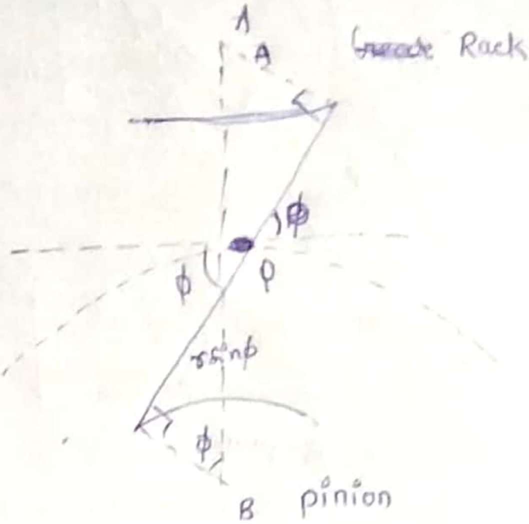
$$\Rightarrow \frac{\cancel{m} t_{min}}{2}\left[\sqrt{1+G(G+2)\sin^2\phi} - 1\right] = \cancel{m} A_p \qquad \left[r = \frac{mt}{2}\right]$$

$$\boxed{t_{min} = \frac{2A_P}{\left[\sqrt{1+G(G+2)\sin^2\phi} - 1\right]}}$$

$$\boxed{T_{min} = \frac{2A_G}{\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1\right]}}$$

Minimum No. of teeth on the pinion in order to avoid interference in involute Rack & Pinion arrangement →





$$\sin\phi = \frac{\text{Addendum (Rack)}}{r\sin\phi}$$

$$r\sin^2\phi = \text{Addendum (Rack)}$$

$$\frac{\cancel{m}\, t_{min}}{2}\cdot\sin^2\phi = \cancel{m}\, A_R$$

$$\boxed{t_{min} = \frac{2A_R}{\sin^2\phi}}$$

→ If Addendum of Pinion & Gear is same →

$$T_{min} = \frac{2A_G}{\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1\right]}$$

Pinion is automatically safe.

$$\boxed{t_{min} = \frac{T_{min}}{G}}$$ Maintain Gear Ratio.

→ If Addendum of Pinion & Gear is different →

$$T_{min} = \frac{2A_G}{\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1\right]}$$

$$t_{min} = \frac{2A_P}{\left[\sqrt{1+G(G+2)\sin^2\phi}-1\right]}$$

$$G = \frac{T_{min}}{t_{min}}$$

Ex:- Two involute gears in a mesh have a velocity ratio of 3. The arc of approach is not to be less than the circular pitch when the pinion is the driver. The pressure angle of the involute teeth is 20°. Determine the least number of teeth on each gear. Also find the addendum of the wheel in terms of module.

Sol:- Given, $\phi = 20^\circ$

$VR = 3$

Arc of approach = Circular Pitch $= \pi m$

Path of approach $= \pi m \cos 20^\circ = 2.952m$

Maximum length of path of Approach $= r \sin\phi$

$= \frac{mt}{2} \sin 20^\circ = 0.171\, mt$

$\therefore \quad 0.171\, mt = 2.952m$

$t = 17.26 \simeq 18$ teeth

$VR = \frac{T}{t} \Rightarrow T = 3 \times 18 \Rightarrow 54$ teeth

Maximum addendum of wheel $= \frac{mT}{2}\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1\right]$

$= \frac{m \times 54}{2}\left[\sqrt{1 + \frac{1}{3}\left(\frac{1}{3}+2\right)\sin^2 20^\circ} - 1\right]$

$= 1.2m$. Ans.

Cycloidal Teeth	Involute Teeth
(a) Pressure angle varies from maximum at beginning of engagement, reduces to zero at the pitch point & again increases to maximum at the end of engagement resulting in less smooth running of the gears.	(a) Pressure angle is constant throughout the engagement of teeth. This results in smooth running of the gears.
(b) It involves double curve for the teeth, epicycloid & hypocycloid. This complicates the manufacture	(b) It involves single curve for the teeth resulting in simplicity of manufacturing of tools.
(c) Owing to difficulty of manufacture, these are costlier.	(c) These are simple to manufacture & thus are cheaper.
(d) Exact centre-distance is required to transmit a constant velocity ratio.	(d) A little variation in the centre distance does not affect the velocity Ratio.

Continue...

Cycloidal Teeth	Involute Teeth
(e) Phenomenon of interference does not occur at all	(e) Interference can occur if the condition of minimum number of teeth on a gear is not followed.
(f) The teeth have spreading flanks and thus are stronger	(f) The teeth have radial flanks & thus are weaker as compared to the cycloidal form for the same pitch.
(g) In this, a convex flank always has contact with a concave face resulting in less wear.	(g) Two convex surfaces are in contact and thus there is more wear.

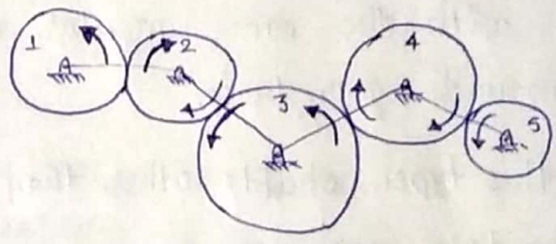
Gear Trains

A gear train is a combination of gears used to transmit motion from one shaft to another.

Types of gear trains:-
1. Simple gear train
2. Compound gear train
3. Reverted gear train
4. Planetary or epicyclic gear train

(1.) Simple Gear train → A series of gears, capable of receiving and transmitting motion from one gear to another is called a simple gear train. In it, all the gear axes remain fixed relative to the frame and each gear is on a separate shaft.

↳ Two external gears of a pair always move in opposite direction.



↳ All odd numbered gears move in one direction and all even numbered gears in the opposite direction.

↳ Speed Ratio, the ratio of the speed of the driving to that of the driven shaft, is negative when the input & output gears rotate in the opposite direction & it is positive when the two ratate in the same direction.

The reverse of speed ratio is known as Train Value of the gear train.

Let, T = No. of Teeth on gear.
N = Speed of a gear in rpm.

$$\frac{N_2}{N_1} = \frac{T_1}{T_2}$$

$$\left[\text{Also, } \frac{\omega_2}{\omega_1} = \frac{2\pi N_2}{2\pi N_1} = \frac{N_2}{N_1}\right]$$

& $\frac{N_3}{N_2} = \frac{T_2}{T_3}$, $\frac{N_4}{N_3} = \frac{T_3}{T_4}$ & $\frac{N_5}{N_4} = \frac{T_4}{T_5}$

On multiplying,

$$\frac{N_2}{N_1} \times \frac{N_3}{N_2} \times \frac{N_4}{N_3} \times \frac{N_5}{N_4} = \frac{T_1}{T_2} \times \frac{T_2}{T_3} \times \frac{T_3}{T_4} \times \frac{T_4}{T_5}$$

$$\boxed{\text{Train Value. } \frac{N_5}{N_1} = \frac{T_1}{T_5} = \frac{\text{No. of Teeth on driving gear}}{\text{No. of Teeth on driven gear}}}$$

Speed Ratio $= \dfrac{1}{\text{Train value}}$

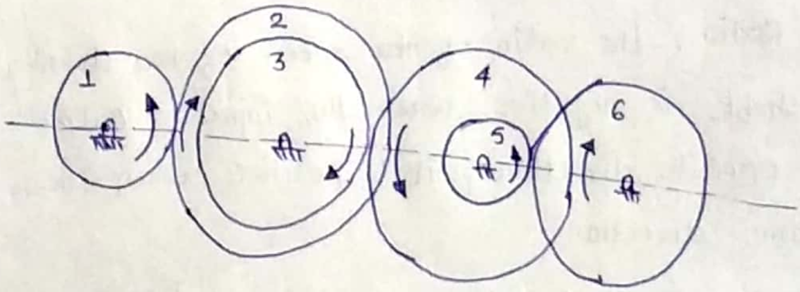
$$\boxed{\frac{N_1}{N_5} = \frac{T_5}{T_1}}$$

Intermediate gears have no effect on speed Ratio, they are known as idlers.

(2.) Compound Gear Train →

when a series of gears are connected in such a way that two or more gears rotate about an axis with the same angular velocity, it is known as compound gear train.

In this type, shaft other than the input & output shafts, carry more than one gear.



$$\frac{N_2}{N_1} = \frac{T_1}{T_2}, \quad \frac{N_4}{N_3} = \frac{T_3}{T_4} \;\&\; \frac{N_6}{N_5} = \frac{T_5}{T_6}$$

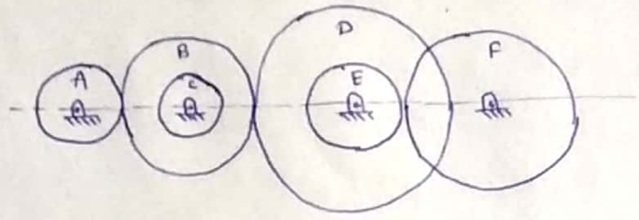
$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} \times \frac{N_6}{N_5} = \frac{T_1}{T_2} \times \frac{T_3}{T_4} \times \frac{T_5}{T_6}$$

$$\frac{N_2}{N_1} \times \frac{N_4}{N_2} \times \frac{N_6}{N_4} = \frac{T_1}{T_2} \times \frac{T_3}{T_4} \times \frac{T_5}{T_6}$$

$$\frac{N_6}{N_1} = \frac{T_1}{T_2} \times \frac{T_3}{T_4} \times \frac{T_5}{T_6}$$

$$\boxed{\text{Train Value} = \frac{\text{Product of No. of Teeth on driving gears}}{\text{Product of No. of Teeth on driven gears}}}$$

Ex:- A compound gear train shown, consist of compound gear B-C and D-E. All gears are mounted on parallel shafts. The motor shaft rotating at 800 rpm is connected to the gear A and the output shaft to the gear F. The number of teeth on gears A, B, C, D, E and F are 24, 56, 30, 80, 32 and 72 respectively. Determine the speed of the gear F.



Sol:-

$$\frac{N_F}{N_A} = \frac{T_A}{T_B} \times \frac{T_C}{T_D} \times \frac{T_E}{T_F}$$

$$\frac{N_F}{800} = \frac{24}{56} \times \frac{30}{80} \times \frac{32}{72}$$

$$N_F = 0.07143 \times 800$$

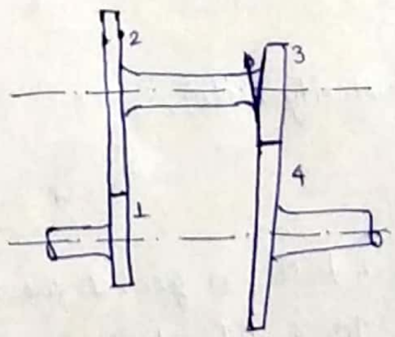
$$N_F = 57.14 \text{ rpm}$$

(3) Reverted Gear Train →

If the axes of the first and the last wheels of a compound gear coinside, it is called a reverted gear train. Used in lathe for back gearing of chuck.

$$\frac{N_4}{N_1} = \frac{\text{Product of No. of teeth on driving gears}}{\text{Product of No. of teeth on driven gears}}$$

$$\frac{N_4}{N_1} = \frac{T_1 \times T_3}{T_2 \quad T_4}$$



Also, if r is the pitch circle radius of a gear

$$r_1 + r_2 = r_3 + r_4$$

Ex:-

Ex:- A reverted gear train is used to provide a speed ratio of 10. The module of gear 1 and 2 is 3.2 mm and of gear 3 and 4 is 2 mm. Determine suitable no. of teeth for each gear. No gear is to have less than 20 teeth. The centre distance b/w shafts is 160 mm. (Assume speed Ratio of gear 1 & 2 = 2.5 & gear 3 & 4 = 4).

Sol:- Given,

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} = 2.5$$

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} = 4$$

Now, $r_1 + r_2 = r_3 + r_4 = 160$

$$\frac{m_1 T_1}{2} + \frac{m_2 T_2}{2} = 160 \qquad \frac{m_3 T_3}{2} + \frac{m_4 T_4}{2} = 160$$

$$3.2(T_1 + T_2) = 320 \qquad 2(T_3 + T_4) = 320$$

$$T_1 + T_2 = 100 \qquad T_3 + T_4 = 160$$

$$T_1 + 2.5T_1 = 100 \qquad T_3 + 4T_3 = 160$$

$$T_1 = 28.57 \qquad \underline{T_3 = 32}$$

$$\underline{T_1 = 28}$$

$$\underline{T_4 = 160 - 32 = 128}$$

$$\underline{T_2 = 100 - 28 = 72}$$

Exact Velocity Ratio,

$$\frac{T_1}{T_2} \times \frac{T_3}{T_4} = \frac{28 \times 32}{72 \times 128} = 10.29 \checkmark$$

(~~close~~ given → 10)

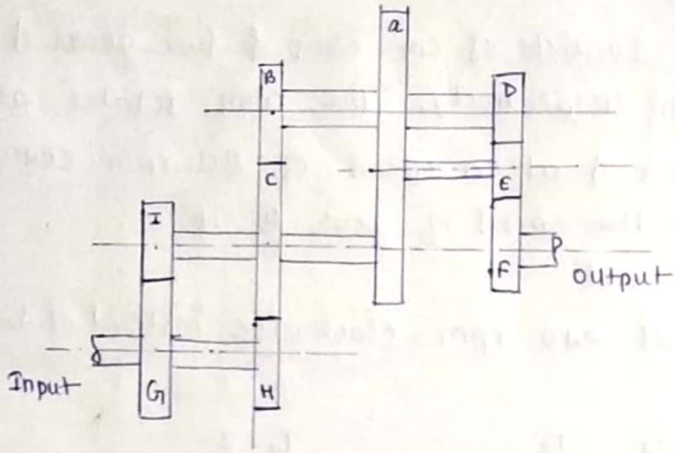
If no. of teeth on gear 1 are taken as 29.

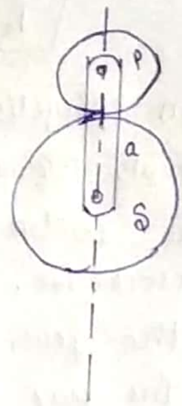
$T_1 = 29, \quad T_2 = 100 - 29 = 71$

the VR, $\frac{T_1}{T_2} \times \frac{T_3}{T_4} = \frac{29 \times 32}{71 \times 128} = 9.79 \checkmark$

Both are correct.

(4) Planetary OR Epicyclic Gear Train → A gear train having a relative motion of axes is called a planetary or epicyclic gear train. In this, the axis of at least one of the gears also moves relative to the frame.





This epicyclic gear consists of wheel B, C, D, E and F and the arm a. Wheel G and H are merely the drivers; G drives the arm a through the wheel I whereas H drives the wheel C.

Analysis → assume that the arm a is fixed. Turn S through x revolutions in the clockwise direction.

revolutions made by $a = 0$

revolutions made by $S = x$

revolutions made by $P = -(T_s/T_p)x$

If the mechanism is locked & system be turned through y revolutions in clockwise direction. Then

revolution made by $a = y$

revolutions made by $S = y + x$

revolutions made by $P = y - \left(\frac{T_s}{T_p}\right)x$

Relative Velocity Method →

Angular velocity of S = angular velocity of S related to a + ang. vel. of a

or $\omega_s = \omega_{sa} + \omega_a$

$N_s = N_{sa} + N_a$

Similarly, $N_p = -N_{pa} + N_a$

~~contd~~ continue ...

$\therefore \quad N_{sa} = N_s - N_a$

$N_{pa} = N_a - N_p$

$$\boxed{\frac{N_{sa}}{N_{pa}} = \frac{N_s - N_a}{N_a - N_p}}$$

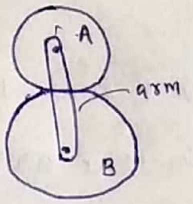
or

$$\boxed{\frac{T_p}{T_s} = -\frac{N_s - N_a}{N_p - N_a}}$$

Ex:- An epicyclic gear train consists of an arm & two gears A and B having 30 and 40 teeth respectively. The arm rotates about the centre of the gear A at a speed of 80 rpm counter-clockwise. Determine the speed of gear B if -

(i) the gear A is fixed and
(ii) the gear A revolves at 240 rpm clockwise instead of being fixed.

Sol:- Consider C.W. → +ve



$\because \quad \frac{\omega_A}{\omega_B} = \frac{T_B}{T_A}$ $\qquad T_A = 30$

$T_B = 40$

$$\boxed{\omega_B = \omega_A \cdot \frac{T_A}{T_B}}$$

Arm motion	A (30)	B (40)
① Let arm is fixed	$+x$	$-x \times \frac{30}{40}$
Arm rotate at y revolution	$y+x$	$y - \frac{3x}{4}$

(i) Gear A is fixed → $y + x = 0$

$x = 80$ (given)

$\boxed{y = -80}$

$\omega_B = y - \frac{3x}{4}$

$\omega_B = -80 - \frac{3 \times 80}{4}$

$\boxed{\omega_B = -140 \text{ rpm}}$ ccw

(ii) $\omega_A = 240$ rpm (given)

$y + x = 240$ $\qquad (y = -80)$

$-80 + x = 240$

$x = 320$ rpm cw

$\omega_B = y - \frac{3x}{4}$

$\omega_B = -80 - \frac{3 \times 320}{4}$

$\omega_B = -80 - 240$

$\omega_B = -320$ rpm ccw

Algebric Method →

(i) $\frac{T_A}{T_B} = -\frac{N_B - N_a}{N_A - N_a}$

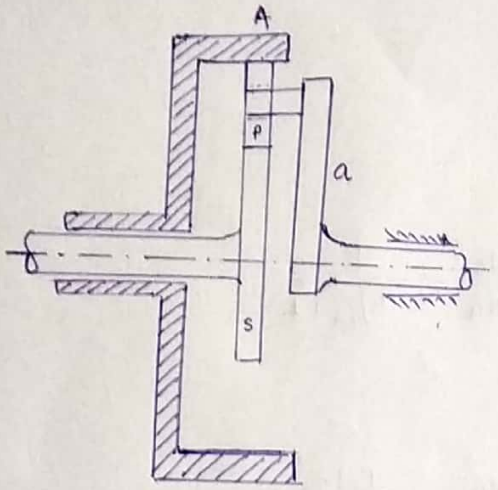
$-\frac{30}{40} = \frac{N_B - 80}{0 - 80}$

$\boxed{N_B = 140 \text{ rpm}}$

(ii) $-\frac{30}{40} = \frac{N_B - 80}{-240 - 80}$

$\boxed{N_B = 320 \text{ rpm}}$

Torque in Epicyclic Trains →



Assume that all the wheels of a gear train rotate at uniform speeds, i.e., acceleration are not involved. Also each wheel is in equilibrium under the action of torques acting on it.

Let N_S, N_a, N_P and N_A be the speed & T_s, T_a, T_P, and T_A the torque transmitted by S, a, P and A respectively.

We have, $T = 0$

$$T_s + T_a + T_P + T_A = 0$$

Now S and a are connected to machinary outside the system and thus transmit external torque. Planet P rotates free (No torque transmission). A is locked by external torque.

$\therefore \quad T_s + T_a + 0 + T_A = 0$

$$T_s + T_a + T_A = 0$$

If A is fixed, T_A is usually known as the breaking or fixing torque. Out of T_s and T_a one will be the driving torque & the other, the output or ressisting torque. Assume no loss in power transmission

$\Sigma T\omega = 0$

or $\Sigma TN = 0$

or $T_s N_s + T_a N_a + T_A N_A = 0$, If A is fixed → $N_A = 0$

$\boxed{T_s N_s + T_a N_a = 0}$

Sun and Planet Gear → (last diagram)

The wheel S and P are generally called the sun and the planet wheels respectively due to analogy of motion of a planet around sun.

If A is fixed, S will be the driving member & if S is fixed, A will be the driving member. In each case the driven member is the arm a.

$$\therefore \quad \boxed{N_a = y = \frac{N_S T_S + N_A T_A}{T_S + T_A}}$$

If the sun wheel S is fixed, $N_S = 0$

Speed of arm

$$N_a = \frac{N_A T_A}{T_S + T_A} \quad \text{or} \quad \frac{N_a}{N_A} = \frac{1}{T_S/T_A + 1}$$

If the annular wheel A is fixed, $N_A = 0$

Speed of arm

$$N_a = \frac{N_S T_S}{T_S + T_A} \quad \text{or} \quad \frac{N_a}{N_S} = \frac{T_S/T_A}{1 + T_S/T_A}$$