(ii) Max. Velocity of the piston.

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 200}{60} = 20.95 \text{ rad/s}.$$

$$\Rightarrow V_p = \omega r \left(\sin\theta + \frac{\sin 2\theta}{2n} \right)$$

$$= 20.95 \times 0.3 \left(\sin 75^\circ + \frac{\sin 150}{(2 \times 3.33)} \right)$$

$$\boxed{V_p = 6.54 \text{ m/s.}}$$

Problem ② The Crank and Connecting rod of a steam engine are 0.3 m and 1.5 m in length. The Crank rotates at 180 rpm. clkwise. Determine the velocity and acceleration of the piston when the Crank is at 40° from the IDC position. Also determine the position of the Crank for zero acceleration of the piston. Using analytical method, determine angular velocity and angular acceleration of the Connecting rod.

Given data:-

$r = 0.3$ m

$l = 1.5$ m

$N = 180$ rpm.

$\theta = 40^\circ$.

(i) Velocity of the piston:

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 180}{60} = 18.85 \text{ rad/s}.$$

$$n = \frac{l}{r} = \frac{1.5}{0.3} = 5$$

$$V_p = \omega r. \left(\sin\theta + \frac{\sin 2\theta}{2n} \right)$$

$$= 18.85 \times 0.3 \times \left(\sin 40^\circ + \frac{\sin 80}{2 \times 5} \right)$$

$$V_p = 4.19 \text{ m/s}.$$

ii. Acceleration of the piston

$$a_p = \omega^2 . r. \left(\cos\theta + \frac{\cos 2\theta}{n} \right).$$

$$= 18.85^2 \times 0.3 \left(\cos 40 + \frac{\cos 80}{5} \right)$$

$$a_p = 85.35 \text{ m/s}^2.$$

iii. Position of the crank for zero acceleration of the piston.

Let θ_1 = Position of crank for $a_p = 0$.

$$a_p = 0.$$

$$\omega^2 . r. \left(\cos\theta_1 + \frac{\cos 2\theta_1}{n} \right) = 0.$$

$n\cos\theta_1 + \cos 2\theta_1 = 0$ $\quad \left\{ \cos 2\theta = 2\cos^2\theta - 1 \right.$

$2\cos^2\theta_1 + 5\cos\theta_1 - 1 = 0$

$$\cos\theta_1 = \frac{-5 \pm \sqrt{5^2 + 4\times 2\times 1}}{2\times 2} = 0.186$$

$$\theta_1 = 79.27^\circ \text{ (or) } 280.73^\circ.$$

iv) Angular velocity of the connecting rod.

$$\omega_{pc} = \frac{\omega\cos\theta}{n}$$

$$= \frac{18.85 \times \cos 40}{5}$$

$$\omega_{pc} = 2.9 \text{ rad/s.}$$

v) Angular acceleration of the connecting rod.

$$\alpha_{pc} = \frac{\omega^2 \sin\theta}{n}$$

$$= \frac{(18.85)^2 \times \sin 40}{5}$$

$$\alpha_{pc} = 45.68 \text{ rad/s}^2$$

Problem ③ The Crank pin circle radius of a horizontal engine is 300 mm. The mass of the reciprocating parts is 250 kg. when the crank has travelled 60° from IDC. the difference b/w the driving and back pressures 0.35 N/mm². the connecting rod length b/w centres is 1.2 m. and the cylinder bore is 0.5 m. If the engine runs at 250 rpm. and if the effect of piston rod diameter is neglected. Calculate

1. Pressure on slide bars.
2. Thrust in the connecting rod.
3. Tangential force on the crank pin
4. Turning moment on the crank shaft.

Given data.

$r = 300 mm.$

$m_R = 250 kg.$

$\theta = 60°$

$P_1 - P_2 = 0.35 N/mm^2.$

$l = 1.2 m.$

$D = 0.5 m$

$N = 250 rpm.$

Now, $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 250}{60} = 26.2 \text{ rad/s}.$

$n = \frac{l}{r} = \frac{1.2}{0.3} = 4.$

Piston effort $F_P = F_L - F_I.$

Net load on the piston $F_L = (p_1 - p_2)\frac{\pi}{4} \times D^2.$

$$= 0.35 \times \frac{\pi}{4} \times 500^2$$

$$F_L = 68730 \text{ N}.$$

Inertia force on reciprocating Parts. F_I

$$F_I = m_R \cdot \omega^2 r \cdot \left(\cos\theta + \frac{\cos 2\theta}{n}\right).$$

$$= 250(26.2)^2 \times 0.3 \left(\cos 60 + \frac{\cos 120}{4}\right)$$

$$F_I = 19306 \text{ N}.$$

Piston effort $F_P = 68730 - 19036 = 49424 \text{ N}.$

1. Pressure on slide bars. F_N.

$$F_N = F_P \tan\phi.$$

where

ϕ = Angle of inclination of the Connecting rod to the line of stroke.

WKT, $\sin\phi = \frac{\sin\theta}{n} = \frac{\sin 60}{4} = 0.216$

$$\phi = 12.5^\circ$$

$F_N = 49424 \times \tan 12.5^\circ$

$F_N = 10960N.$

2. Thrust in the Connecting rod. F_Q.

$$F_Q = \frac{F_P}{\cos\phi}.$$

$$= \frac{49424}{\cos 12.5^\circ}$$

$F_Q = 50620\ N.$

3. Tangential force on the crank pin.

$F_T = F_Q \sin(\theta + \phi)$

$= 50620 \sin(60 + 12.5)$

$F_T = 48,280\ N.$

4. Turning moment on the crank shaft.

$T = F_T \times r.$

$= 48280 \times 0.3$

$T = 14484\ N.m.$

Problem ④ The crank and connecting rod of a petrol engine, running at 1800 rpm. are 50mm and 200mm respectively. The diameter of piston is 80mm and the mass of the reciprocating parts is 1 kg. At a point during the power stroke, the pressure on the piston is 0.7 N/mm². When it has moved 10mm from the IDC. Determine 1. Net load on the gudgeon pin.

2. Thrust in the connecting rod.

3. Reaction b/w piston & cylinder.

4. The engine speed at which the above values become zero.

Given data:

$N = 1800$ rpm.

$r = 50$ mm

$l = 200$ mm.

$D = 80$ mm

$m_R = 1$ kg.

$p = 0.7$ N/mm².

$x = 10$ mm

Now,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1800}{60} = 188.5 \text{ rad/s}.$$

1. Net load on the gudgeon pin F_p.

$$F_p = F_L - F_I.$$

net load on the piston $F_L = \frac{\pi}{4} D^2 \times p$

$$= \frac{\pi}{4} \times 80^2 \times 0.7$$

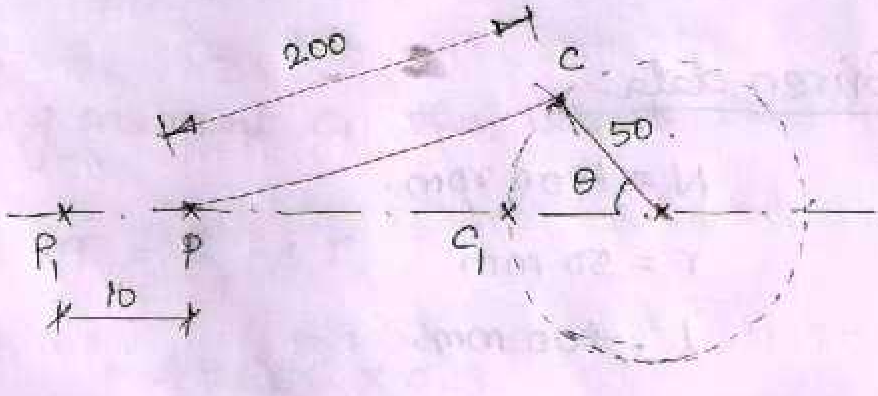
$$F_L = 3520 \text{ N}.$$

Inertia force on the reciprocating parts. F_I

$$F_I = m_R \cdot a_R.$$

$$F_I = m_R \cdot \omega^2 \cdot r \cdot \left(\cos\theta + \frac{\cos 2\theta}{n} \right).$$

where $n = \frac{l}{r} = \frac{200}{50} = 4.$



By measurement, we find that $\theta = 33^\circ$.

$$F_I = 1 \times (188.52)^2 \times 0.05 \left(\cos 33 + \frac{\cos 66}{4} \right)$$

$$F_I = 1671 \text{ N}.$$

net load on the gudgeon pin

$$F_p = F_L - F_I$$

$$= 3520 - 1671$$

$$F_p = 1849\ N.$$

2. Thrust in the Connecting rod. F_Q

WKT,

$$\sin\phi = \frac{\sin\theta}{n} = \frac{\sin 33}{4} = 0.1361$$

$$\Rightarrow \phi = 7.82^\circ.$$

now,

$$F_Q = \frac{F_p}{\cos\phi}$$

$$= \frac{1849}{\cos 7.82}$$

$$F_Q = 1866.3\ N.$$

3. Reaction b/w the piston and cylinder.

$$F_N = F_p \tan\phi$$

$$= 1849 \times \tan 7.82$$

$$F_N = 254\ N.$$

4. Engine speed at which the above values will become zero:

$$F_p = 0!$$

$$F_L = F_I.$$

Let ω_1 be the speed in rad/s at $F_p = 0$.

$$m_R \omega_1^2 r \left(\cos\theta + \frac{\cos 2\theta}{n}\right) = \frac{\pi}{4} D^2 \times p.$$

$$1. \omega_1^2 \times 0.05 \left(\cos 33 + \frac{\cos 66}{4}\right) = \frac{\pi}{4} \times 80^2 \times 0.7$$

$$\Rightarrow \omega_1 = 273.6 \text{ rad/s.}$$

Corrosponing speed in rpm.

$$N_1 = 2612 \text{ rpm}.$$

Problem ⑤

A Vertical Petrol engine 100mm diameter and 120 mm stroke has a Connecting rod 250mm long. The mass of the piston is 1.1 kg. The Speed is 2000 rpm. on the expansion stroke with a crank 20° from TDC, the gas pressure is 700 kN/m².

Determine. 1. Net force on the piston

2. Resultant load on gudgeon pin.

3. Thrust on Cylinder walls.

and 4. Speed above which, other things remaining same, the gudgeon pin load would be reversed in direction.

Given Data:

$D = 100\ mm$

$L = 120\ mm$

$r = \frac{L}{2} = 60\ mm$

$l = 250\ mm$

$m_R = 1.1\ kg.$

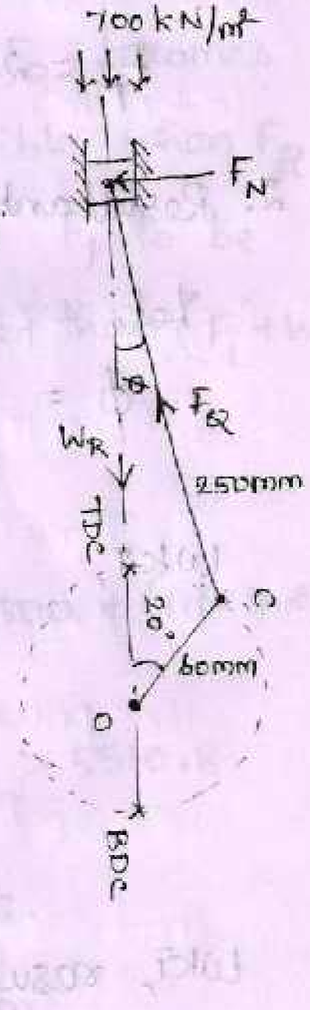
$N = 2000\ rpm.$

$\theta = 20^\circ$

$p = 700\ kN/m^2.$

i. Net force on the piston.

The configuration diagram of a vertical engine shown in fig.



WKT,

Force due to gas pressure.

$$F_L = p \times \frac{\pi}{4} \times D^2$$

$$= 700 \times 10^3 \times \frac{\pi}{4} \times 0.1^2$$

$$= 5500 N$$

$$n = \frac{l}{r} = \frac{0.25}{0.06} = 4.17$$

Inertia force on the piston.

$$F_I = m_R \cdot \omega^2 \cdot r \left(\cos\theta + \frac{\cos 2\theta}{n} \right)$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 2000}{60} = 209.5\ rad/s.$$

$$F_I = 1.1 \times (209.5)^2 \times 0.06 \times \left(\cos 20^\circ + \frac{\cos 40^\circ}{4.17} \right)$$

$$= 3254 \text{ N}$$

WKT, for a vertical engine.

Net force on the piston.

$$F_P = F_L - F_I + W_R.$$

$$= F_L - F_I + (m_R \cdot g).$$

$$= 5500 - 3254 + (1.1 \times 9.81)$$

$$F_P = 2256.8 \text{ N}.$$

2. Resultant load on the gudgeon pin.

Let

ϕ = Angle of inclination of the connecting rod to the line of stroke

WKT,

$$\sin \phi = \frac{\sin \theta}{n}$$

$$\phi = \sin^{-1} \left(\frac{\sin 20^\circ}{4.17} \right)$$

$$\phi = 4.7^\circ$$

WKT, resultant load on the gudgeon pin

$$F_Q = \frac{F_P}{\cos \phi} = \frac{2256.8}{\cos 4.7^\circ}$$

$$F_Q = 2265 \text{ N}.$$