DYNAMICS OF MACHINES.

UNIT I FORCE ANALYSIS.

Dynamic force analysis - Inertia force and Inertia Torque - D Alembert's Principle - Dynamic analysis in reciprocating engines - Gas forces - Inertia effect of Connecting rod - Bearing loads - Crank Shaft torque - Turning moment diagrams - Fly wheels - Fly wheels of punching Presses - Dynamics of Cam-follower mechanism.

Ref : 'Theory of Machines' 'R.S. KHURMI.' chapter 15 & 16 (Pg. No. 514 - 611).

Inertia force:

The Inertia force is an imaginary force, which when acts upon a rigid body, brings it in an equilibrium position. It is numerically equal to the accelerating force in magnitude but opposite in direction.

Mathematically

$$\text{Inertia force} = -\text{Accelerating force}$$
$$= -m.a.$$

where,

m = Mass of the body, and

a = Linear acceleration of the centre of gravity of the body.

Inertial Torque:-

The Inertia torque, which when applied upon the rigid body, brings it in equilibrium position. It is equal to the accelerating couple in magnitude but opposite in direction.

D - Alembert's Principle.

"The resultant force acting on a body together with the reversed effective force (or inertia force) are in equilibrium."

According to Newton's Second law of motion

$$F = m.a. \qquad \text{--- (1)}$$

where F = resultant force acting on Body.

m = mass of the body.

a = linear acceleration of the Centre of mass of the body.

the equation (1) may also be written as

$$F - ma = 0. \qquad \text{--- (2)}$$

If the quantity $-m.a.$ be treated as a force, equal, opposite and with the same line of action. as the resultant force F, and include this force with the System of forces of which 'F' is the resultant, then the Complete System of forces will be in equilibrium. This Principle is known as 'D-Alembert's principle.

The equal and opposite force $-ma$ is known as reversed effective force or the inertia force (F_I). The equation ② may be written as

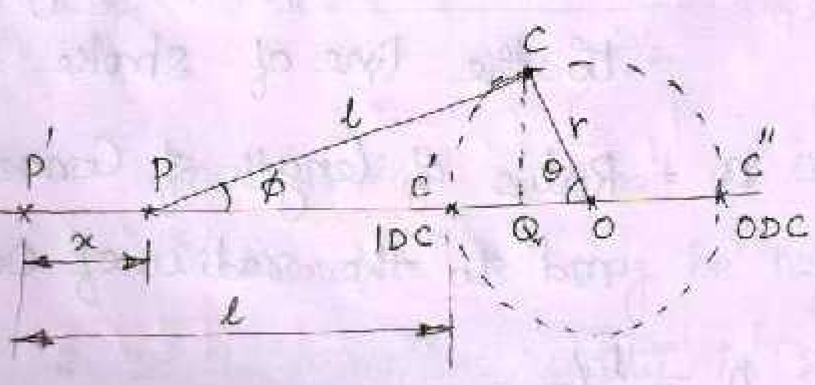
$$F + F_I = 0.$$

This principle is used to reduce a dynamic problem into an equivalent static problem.

Velocity and Acceleration of the reciprocating Parts in Engines.

1. Analytical method
2. Graphical method.
   i) Klien's Construction.
   ii) Ritterhaus's Construction.
   iii) Bennett's Construction.

Analytical method for velocity and Acceleration of the piston



Consider the motion of a crank and connecting rod of a reciprocating steam engine as shown in fig. Let OC be the crank and PC the connecting rod.

Let the crank rotates with angular velocity of 'ω' rad/s. and the crank turns through an angle 'θ' from inner dead centre (IDC). Let 'x' be the displacement of a reciprocating body P from IDC. after time 't' seconds, during which the crank has turned through an angle 'θ'.

Velocity of the piston:

l = Length of connecting rod between the centres

r = Radius of crank ~~or~~

ϕ = Inclination of connecting rod to the line of stroke PO.

n = Ratio of length of connecting rod to the radius of crank

$n = l/r$.

Velocity of the piston (v_{po})

$$v_{po} = v_p = \omega . r . \left(\sin\theta + \frac{\sin 2\theta}{2n} \right).$$

Acceleration of the piston (a_p)

$$a_p = \omega^2 . r . \left(\cos\theta + \frac{\cos 2\theta}{n} \right).$$

Note : i) when crank is at IDC, $\theta = 0°$

ii) when crank is at ODC, $\theta = 180°$

Angular velocity and Acceleration of the Connecting rod.

Angular velocity of the connecting rod PC.

$$\omega_{pc} = \frac{\omega \cos\theta}{(n^2 - \sin^2\theta)^{1/2}}$$

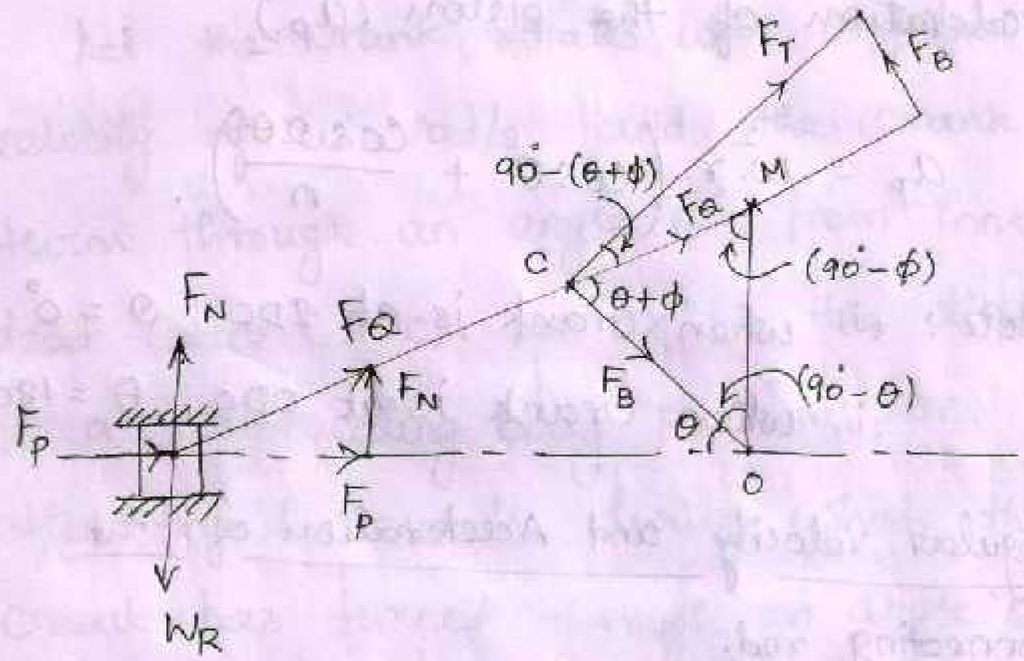
Angular Acceleration of the Connecting rod.

$$\alpha_{pc} = \frac{-\omega^2 \sin\theta \, (n^2 - 1)}{(n^2 - \sin^2\theta)^{3/2}}.$$

Note :- 1. Since $\sin^2\theta$ is small as compared to n^2 therefore it may be neglected.

2. In α_{pc} equation, unity is small as compared to n^2. hence unity may neglected.

Forces on the Reciprocating parts of an Engine, neglecting the weight of the Connecting rod.



m_R = Mass of the reciprocating parts. (piston, Crosshead pin or gudgeon pin).

W_R = Weight of reciprocating parts ($m_R \cdot g$).

1. Piston Effort. (F_P)

F_P = Net load on the piston $\pm$ Inertia force. − Frictional resistance.

$$F_P = F_L \pm F_I - R_F$$

when the piston is accelerated (−ve sign) and the piston is retarded (+ve sign).

Net load on piston F_L

$$F_L = p_1 A_1 - p_2 A_2$$

$$= p_1 A_1 - p_2 (A_1 - a).$$

$p_1 A_1$ = Pressure and Crossectional area on the back end side of piston

$p_2 A_2$ = pressure and crossectional area on the Crank end side of piston.

a = Crossectional area of piston rod.

2. Force acting along the Connecting rod. (F_Q)

$$F_Q = \frac{F_P}{\cos\phi}$$

$$= \frac{F_P}{\left(1 - \frac{\sin^2\theta}{n^2}\right)^{1/2}}$$

3. Thrust on the sides of the cylinder walls (or) Normal reaction on the guide bars. (F_N)

$$F_N = F_Q \sin\phi.$$

$$= F_P \tan\phi.$$

4. Crank-pin effect and thrust on Crank shaft bearings.

The Force acting on the Connecting rod F_Q may be resolved into two Components, one Perpendicular to the Crank and the other along the Crank.

The Component of F_Q $\perp^r$ to the Crank is known as Crank pin effort. (F_T). The Component of F_Q along the crank produces a thrust on the Crank shaft bearings (F_B).

$$F_T = F_Q \sin(\theta + \phi)$$

$$= \frac{F_P}{\cos\phi} \sin(\theta + \phi)$$

$$F_B = F_Q \cos(\theta + \phi)$$

$$= \frac{F_P}{\cos\phi} \times [\cos(\theta + \phi)].$$

5. Crank effort (or) Turning moment (or) Torque on the crank shaft. (T).

The product of the Crank pin effort (F_T) and the Crank pin radius (r) is known as Crank effort.

Crank effort $T = F_T \cdot r$.

$$= F_p (\sin\theta + \cos\theta . \tan\phi) . r$$

$$T = F_p \times r \left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}} \right).$$

Problem ①. If the Crank and the Connecting rod are 300 mm and 1 m long respectively and the Crank rotates at a Constant speed of 200 rpm. determine i) The Crank angle at which the maximum velocity occurs and ii) maximum velocity of the piston.

Given Data:-

$r = 300$ mm

$l = 1000$ mm

$N = 200$ rpm.

To Find:-

i) 'θ' at 'V_p' max.

ii) V_p max = ?

(i) Crank angle at which the maximum velocity occurs.

Let θ = Crank angle from IDC at which the max. velocity occurs.

$$n = \frac{l}{r} = \frac{1000}{300} = 3.33$$

Velocity of piston.

$$V_p = \omega r \left(\sin\theta + \frac{\sin 2\theta}{2n} \right).$$

For max. velocity of the piston.

$$\frac{dV_p}{d\theta} = 0.$$

$$\Rightarrow \frac{d\left[\omega r \left(\sin\theta + \frac{\sin 2\theta}{2n} \right)\right]}{d\theta} = 0.$$

$$\Rightarrow \omega r \left(\cos\theta + \frac{2\cos 2\theta}{2n} \right) = 0.$$

$$\Rightarrow n\cos\theta + 2\cos^2\theta - 1 = 0 \quad \left\{ \cos 2\theta = 2\cos^2\theta - 1 \right\}$$

$$\Rightarrow 2\cos^2\theta + 3.33\cos\theta - 1 = 0.$$

$$\cos\theta = \frac{-3.33 \pm \sqrt{3.33^2 + (4\times 2\times 1)}}{2\times 2} = 0.26$$

$$\Rightarrow \boxed{\theta = 75^\circ.}$$